Parallel Programming
Exercise Session 6

Spring 2025

Today

Post-Discussion Ex. 5 (+ some theory)

- Ex.5 Theory Tasks
- Ex.5 Programming Tasks

Theory

Pre-Discussion Ex. 6

Big Kahoot

Theory Recap based on Kahoot results

Exam Preparation Session

Monday, March 31, 11:15 – 12:00

Tuesday, April 1, 10:15 – 12:00

HG F 5 / HG F 7

Hosted by ??? / Vera Schubert and Jackson Stanhope

Theory + Post-Discussion Ex. 5

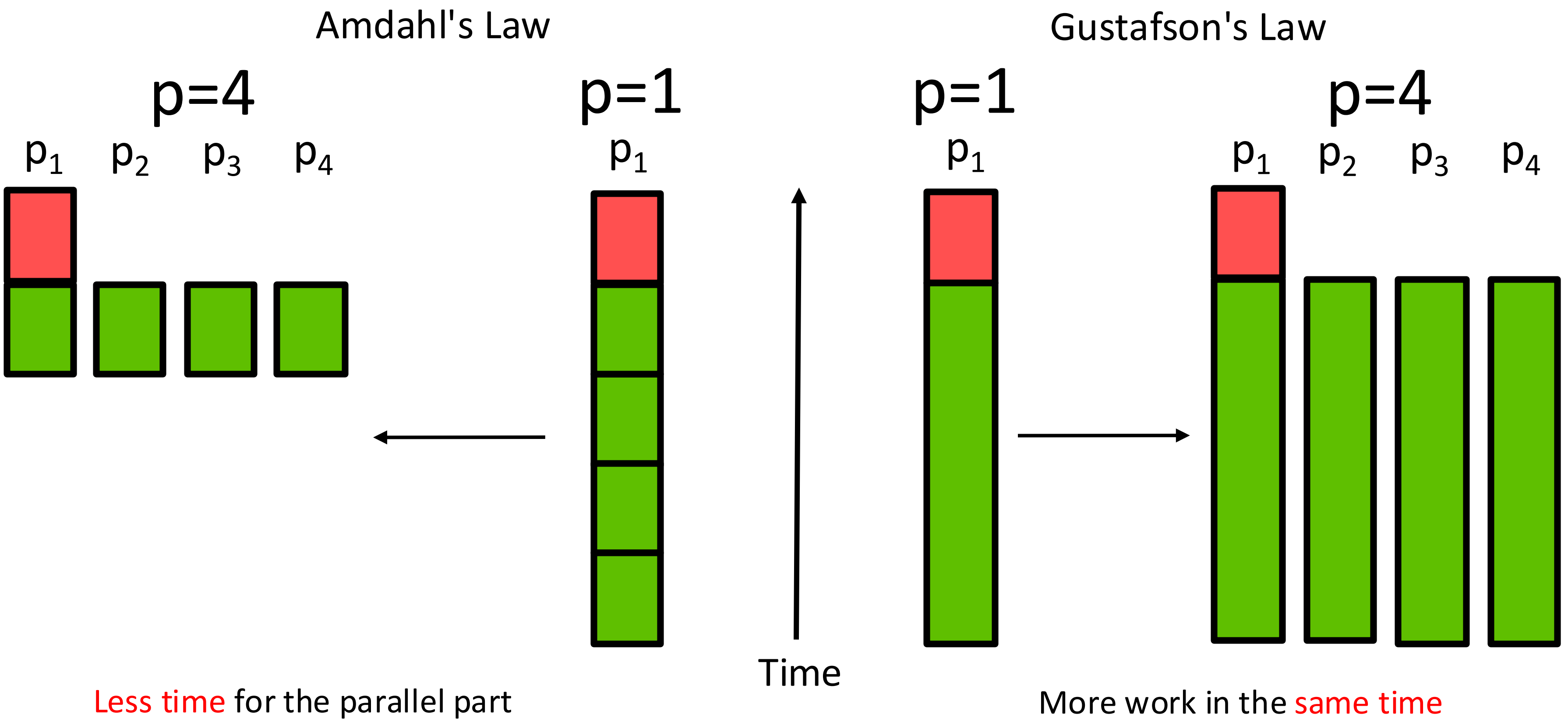
Ex. 5 Theory Tasks

Recall: Amdahl's vs Gustafson's Law

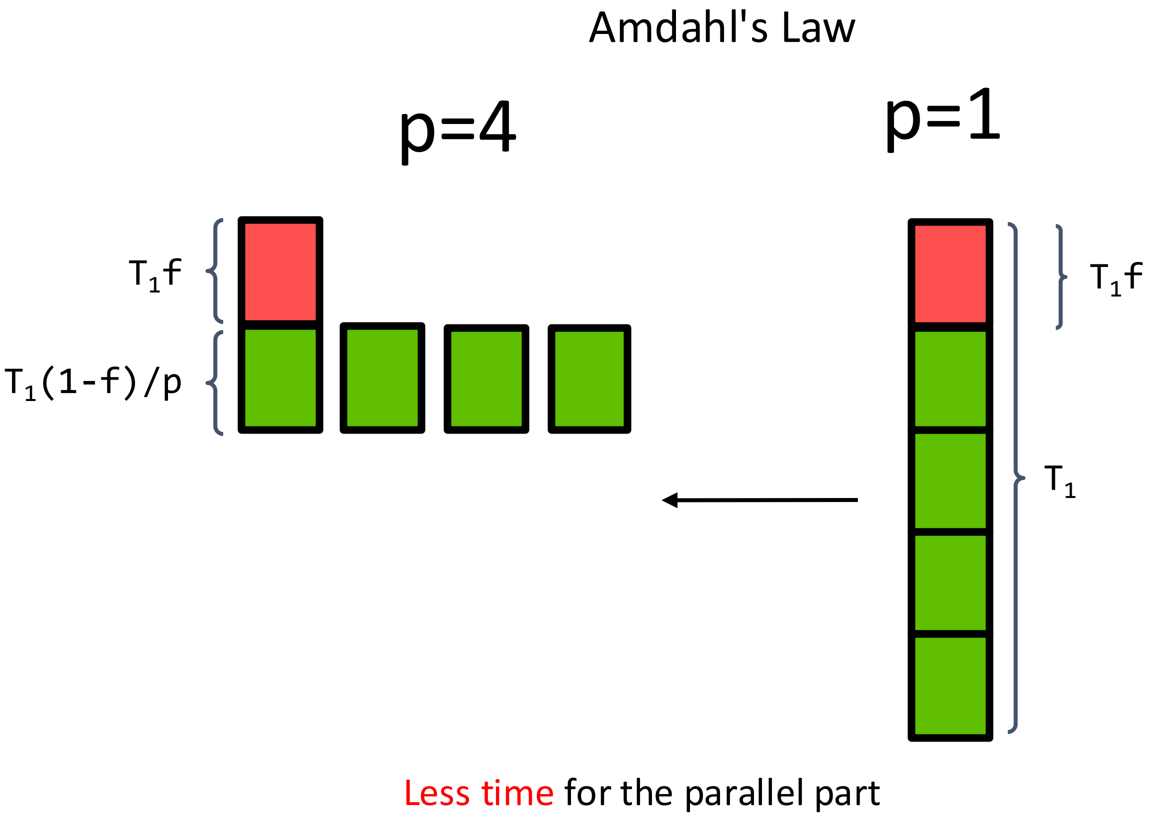
The key goal is to:

- ➔ Understand the main difference and implications (i.e., when to use which formula)
- ➔ Know how to derive the formulas based on your understanding, not because you memorized them for the exam

Recall: Amdahl's vs Gustafson's Law



Amdahl's Law Derivation



T_1 - sequential time

f - sequential fraction

T_p - parallel time on p processors

$$T_p = T_1 f + \frac{T_1(1-f)}{p}$$

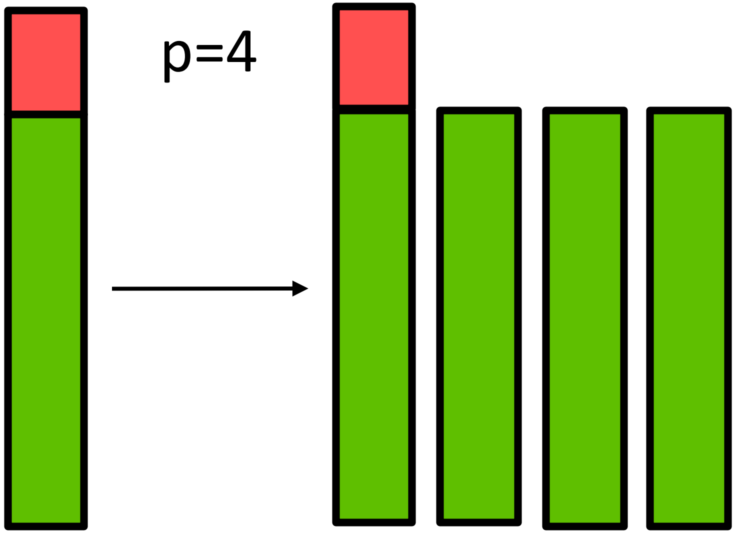
S_p - speedup

$$S_p = \frac{T_1}{T_p}$$

$$S_p = \frac{T_1}{T_1 f + \frac{T_1(1-f)}{p}} = \frac{1}{f + \frac{(1-f)}{p}}$$

Gustafson's Law Derivation

Gustafson's Law



More work in the same time

W - Work with 1 processor

W_p - Work with p processors

f - sequential fraction

$$W_1 = W_1 f + W_1(1-f)$$

$$W_p = W_1 f + W_1(1-f)p$$

S_p - speedup

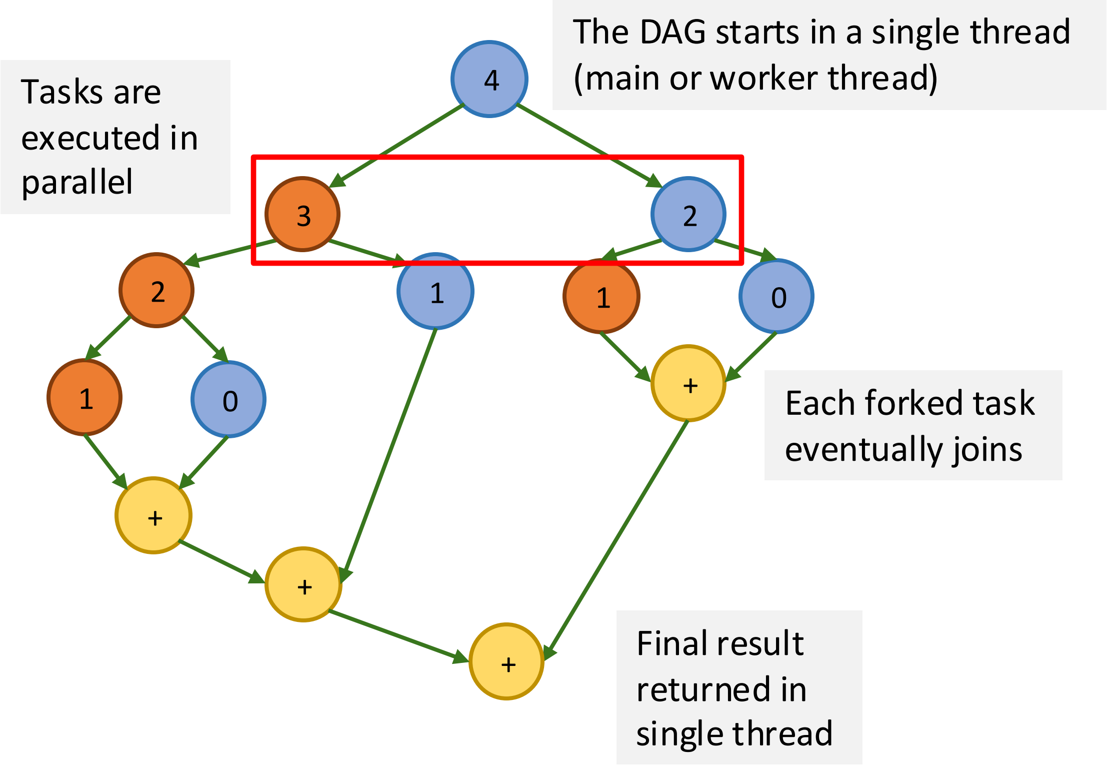
$$S_p = W_p/W_1$$

$$S_p = f + (1-f)p$$

fib(4) task graph

```
public class Fibonacci {
    public static long fib(int n) {
        if (n < 2) {
            return n;
        }
        spawn task for fib(n-1);
        spawn task for fib(n-2);
        wait for tasks to complete
        return addition of task results
    }
}
```

fib(4) task graph FJ




```
public class Fibonacci {
    public static long fib(int n) {
        if (n < 2) {
            return n;
        }
        spawn task for fib(n-1);
        spawn task for fib(n-2);
        wait for tasks to complete
        return addition of task results
    }
}
```

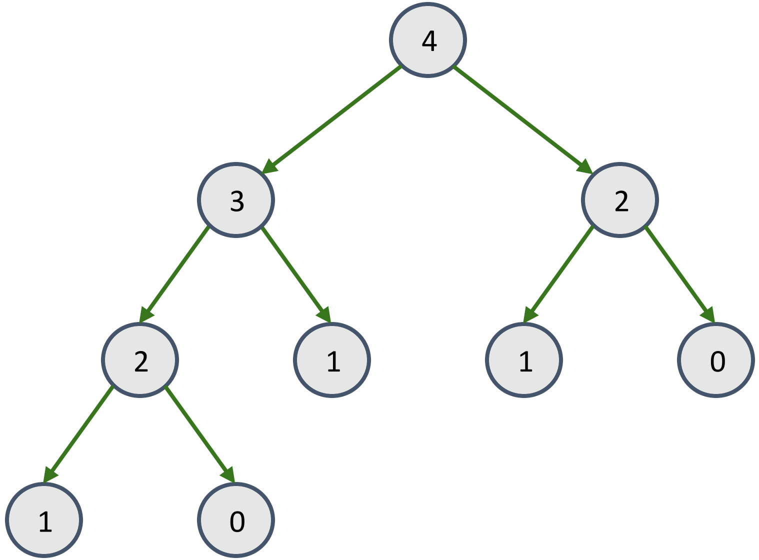
What is a task?

`new forked task`, `continuation of current task`, `join`

What is an edge?

`spawn, same procedure, wait`

fib(4) simplified task graph




```
public class Fibonacci {
    public static long fib(int n) {
        if (n < 2) {
            return n;
        }
        spawn task for fib(n-1);
        spawn task for fib(n-2);
        wait for tasks to complete
        return addition of task results
    }
}
```

What is a task?

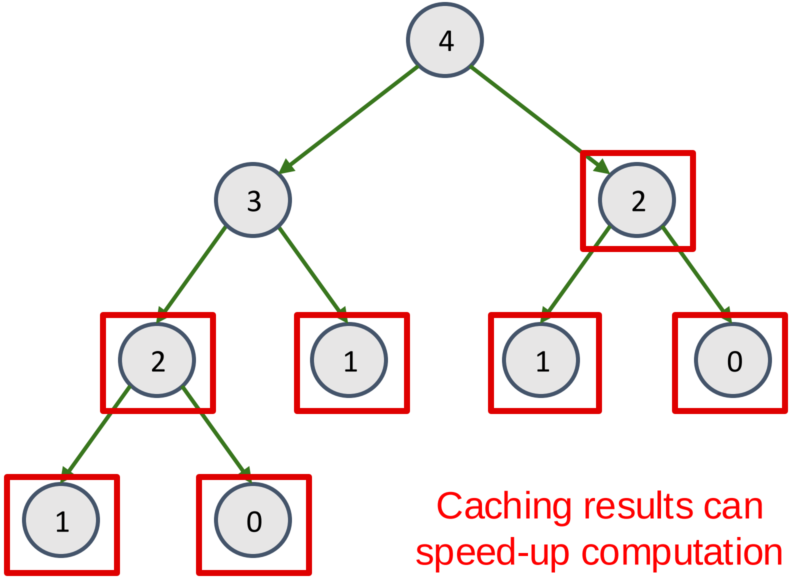
`Call to Fibonacci`

What is an edge?

`spawn`
`(no dependency within same procedure)`

Simpler at the expense of not modelling joins and inter-process dependencies

fib(4) simplified task graph



Simpler at the expense of not modelling joins and inter-process dependencies

```
public class Fibonacci {
    public static long fib(int n) {
        if (n < 2) {
            return n;
        }
        spawn task for fib(n-1);
        spawn task for fib(n-2);
        wait for tasks to complete
        return addition of task results
    }
}
```

What is a task?

`Call to Fibonacci`

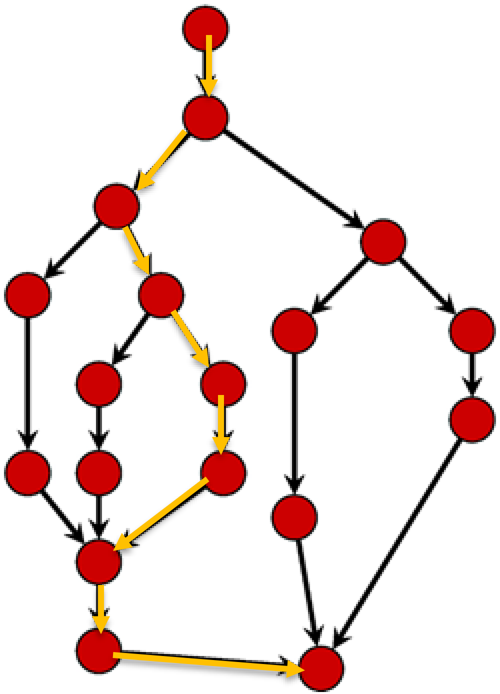
What is an edge?

`spawn`
`(no dependency within same procedure)`

How would you memoize in this example?

- Shared array (initialize with -1)
- We don‘t need to synchronize since different threads will write the same value to the same entry
- Reading the values is no problem

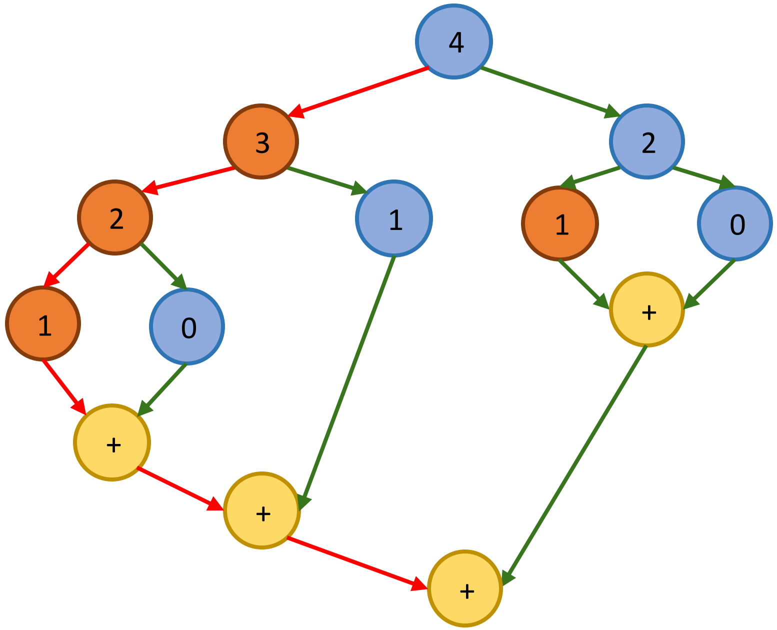
Task Graphs



Critical path: path from start to end that takes the longest (for some metric)

Example: #nodes

fib(4) task graph FJ



critical path length is 7 tasks

```
public class Fibonacci {
    public static long fib(int n) {
        if (n < 2) {
            return n;
        }
        spawn task for fib(n-1);
        spawn task for fib(n-2);
        wait for tasks to complete
        return addition of task results
    }
}
```

What is a task?

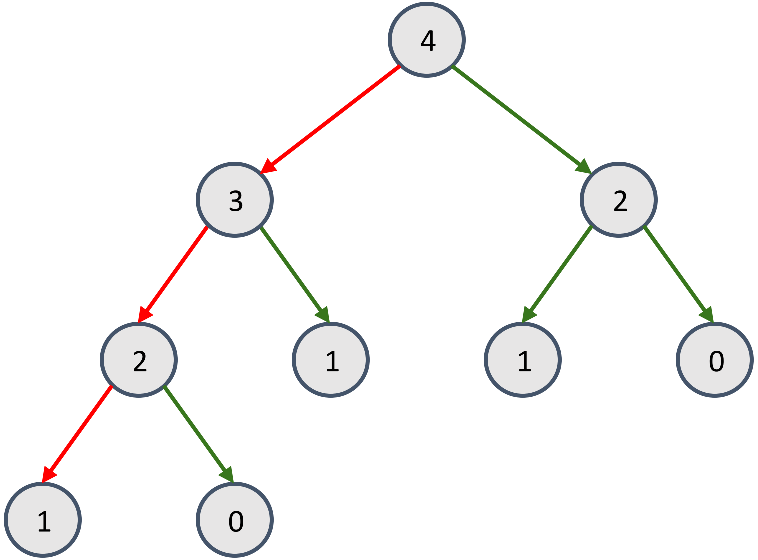
new forked task, continuation of current task, join

What is an edge?

spawn, same procedure, wait

fib(4) simplified task graph

```
public class Fibonacci {
    public static long fib(int n) {
        if (n < 2) {
            return n;
        }
        spawn task for fib(n-1);
        spawn task for fib(n-2);
        wait for tasks to complete
        return addition of task results
    }
}
```



critical path length is 4 tasks

What is a task?

`Call to Fibonacci`

What is an edge?

`spawn`
`(no dependency within same procedure)`

Task Graph Simplified

Task: Call to add()

Cut-off: 1

Adding eight numbers:

$$\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{1+2}_{+}+3}_{+}+4}_{+}+5}_{+}+6}_{+}+7}_{+}+8}_{+}$$

Task Graph Simplified

Task: Call to add()

Cut-off: 1

Adding eight numbers:

$$\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{1+2}_{+}+3}_{+}+4}_{+}+5}_{+}+6}_{+}+7}_{+}+8}_{+}$$

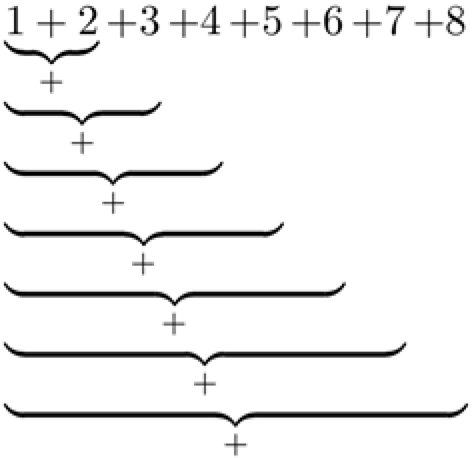
What is the corresponding task graph?

Task Graph Simplified

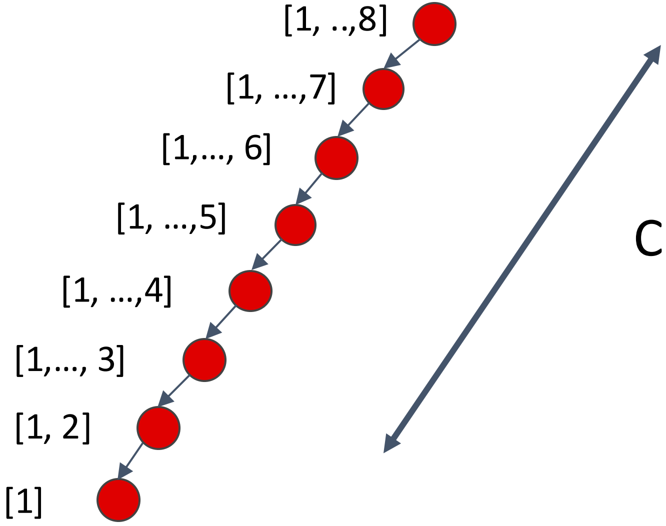
Task: Call to add()

Cut-off: 1

Adding eight numbers:



What is the corresponding task graph?



Critical path

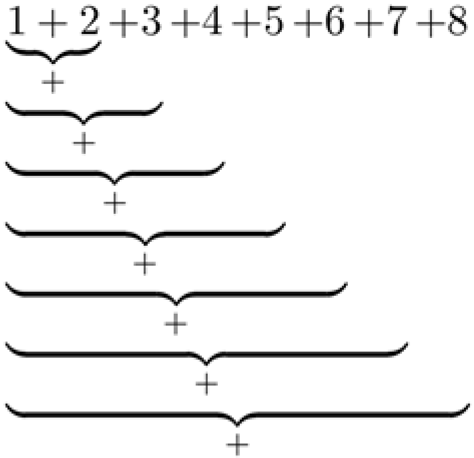
8

Task Graph FJ

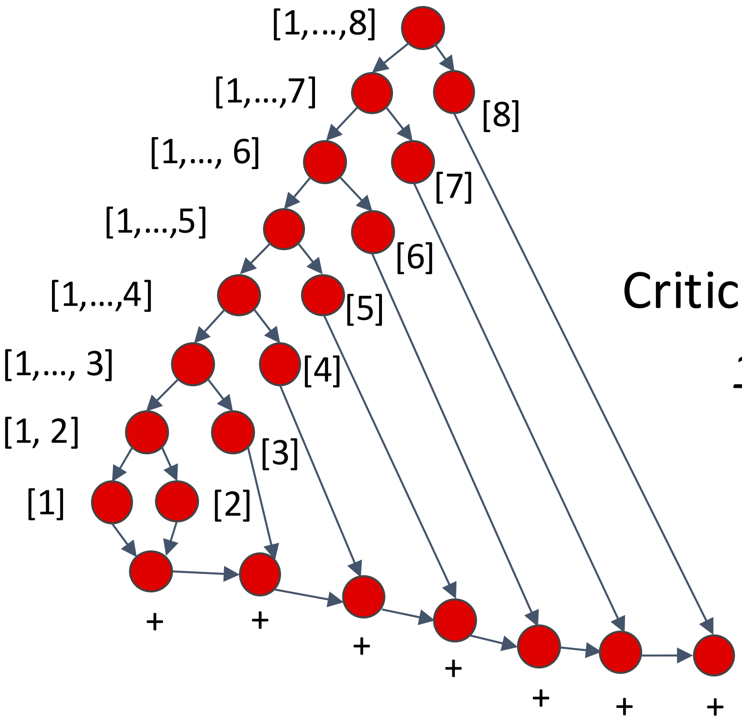
Task: fork, join, continuation

Cut-off: 1

Adding eight numbers:



What is the corresponding task graph?



Critical path

15

Task Graph Simplified

Task: Call to add()

Cut-off: 1

Adding eight numbers:

$$\underbrace{\underbrace{\underbrace{1+2}_{+}+\underbrace{3+4}_{+}}_{+}+\underbrace{\underbrace{5+6}_{+}+\underbrace{7+8}_{+}}_{+}}_{+}$$

What is the corresponding task graph?

Task Graph Simplified

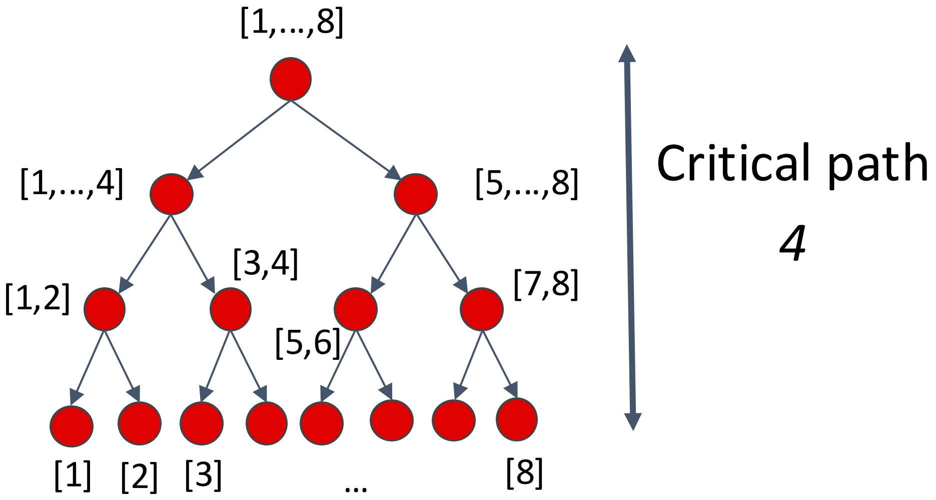
Task: Call to add()

Cut-off: 1

Adding eight numbers:

$$\underbrace{\underbrace{\underbrace{1+2}_{+}+\underbrace{3+4}_{+}}_{+}+\underbrace{\underbrace{5+6}_{+}+\underbrace{7+8}_{+}}_{+}}_{+}$$

What is the corresponding task graph?



Task Graph FJ

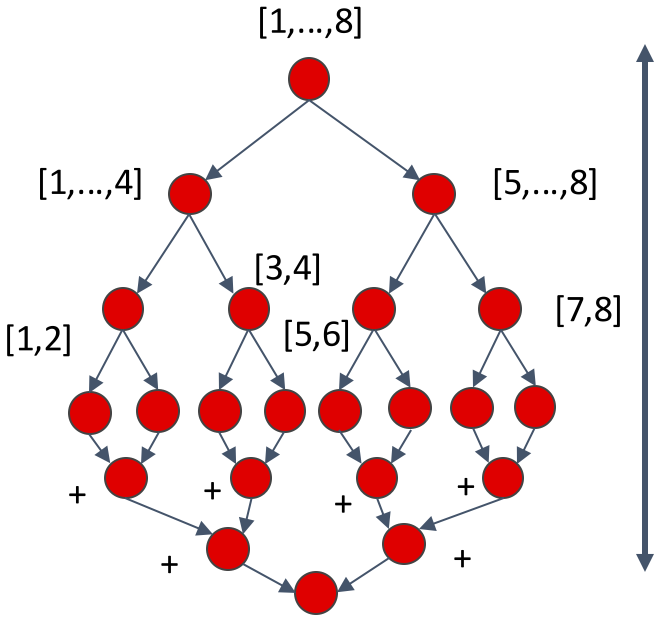
Task: fork, join, continuation

Cut-off: 1

Adding eight numbers:

$$\underbrace{\underbrace{\underbrace{1+2}_{+}+\underbrace{3+4}_{+}}_{+}+\underbrace{\underbrace{5+6}_{+}+\underbrace{7+8}_{+}}_{+}}_{+}$$

What is the corresponding task graph?



Critical path

7

Task Graphs

A wide task graph → higher potential parallelism

A deep task graph → more sequential dependencies

„Easy“ points

- Usually tasks about Amdahl‘s / Gustafson‘s Law, Pipelining, Task Graphs are the easy tasks in the exam
- You can definitely collect about ~25% of the points by solving those tasks.
- Practicing for those tasks is straightforward
  - Get familiar with the differences of Amdahl and Gustafson. Be able to derive the formulas by yourself
  - Understand formulas intuitively: Amdahl, Gustafson, Pipelining
  - Practice Task Graphs

Ex. 5 Programming Tasks

Task 1: Search And Count

Search an array of integers for a certain feature and count integers that have this feature:

- Light workload: count number of non-zero values.
- Heavy workload: count how many integers are prime numbers.

We will study single threaded and multi-threaded implementation of the problem.

Task 1 A: Search And Count - Sequential

```
public class SearchAndCountSingle {
  private int[] input;
  private Workload.Type type;

  private SearchAndCountSingle(int[] input, Workload.Type wt) {
    this.input = input;
    this.type = wt;
  }

  private int count() {
    int count = 0;
    for (int i = 0; i < input.length; i++) {
      if (Workload.doWork(input[i], type)) count++;
    }
    return count;
  }
}
```

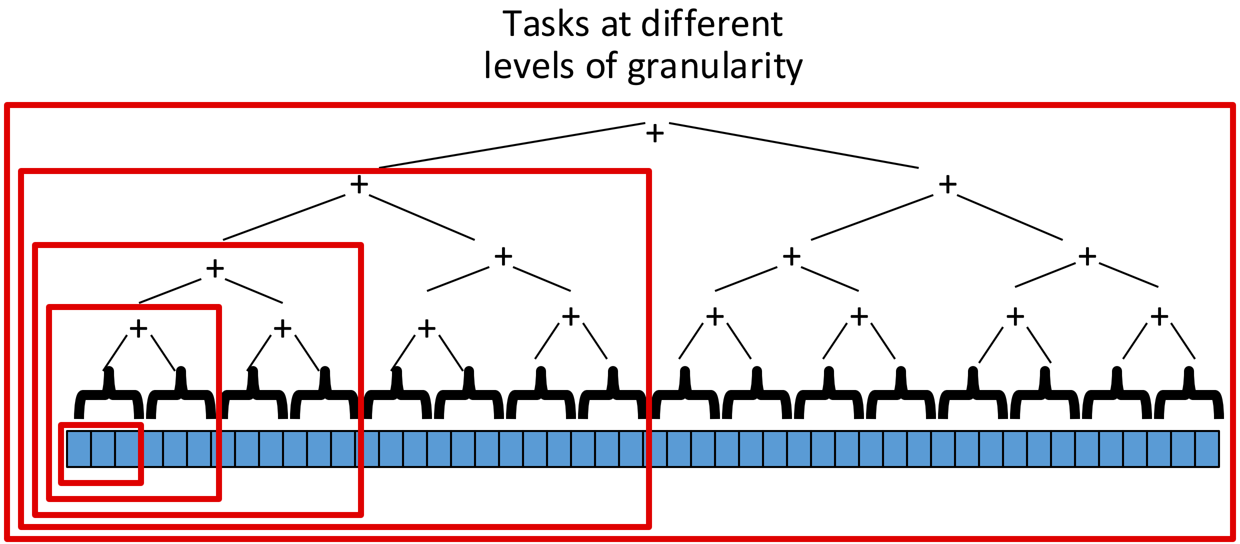
Straightforward implementation. Simply iterate through the input array and count how many times given event occurs.

Divide and Conquer

Basic structure of a divide-and-conquer algorithm:

1. If problem is small enough, solve it directly
2. Otherwise
    a. Break problem into subproblems
    b. Solve subproblems recursively
    c. Assemble solutions of subproblems into overall solution

Divide and Conquer



What determines a task?

i) input array

ii) start index

iii) length/end index

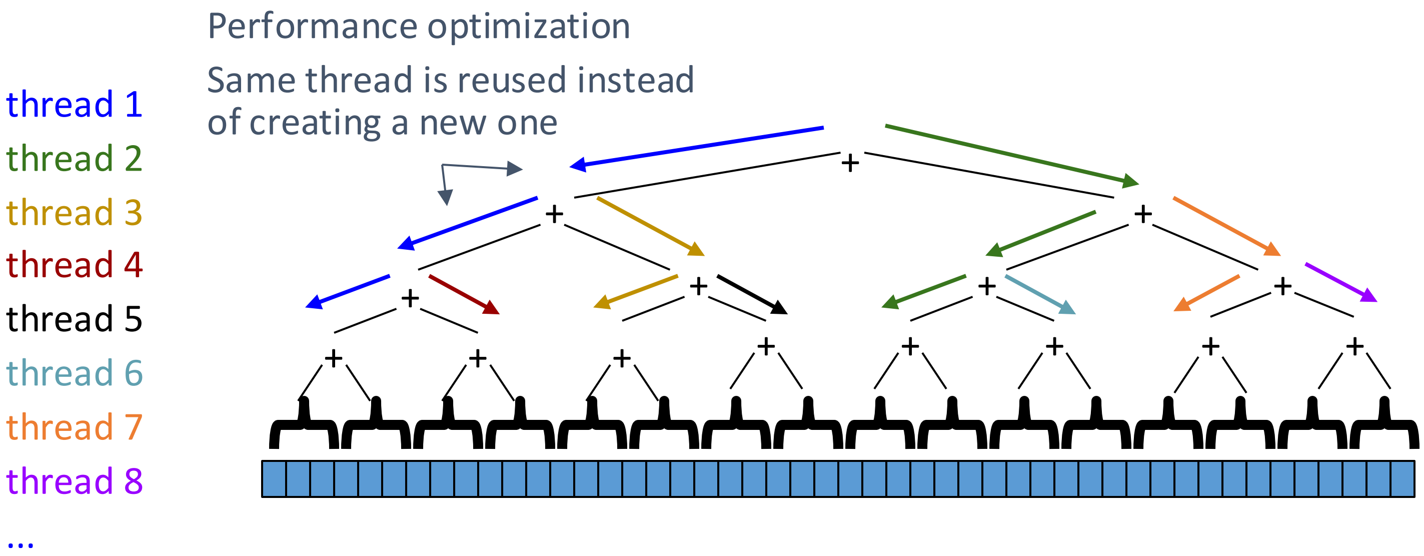
These are fields we want to store in the task

Task B

SearchAndCountThreadDivideAndConquer.java

- → Divide and conquer
- → Do not create more threads than numThreads

Divide and Conquer Parallelization

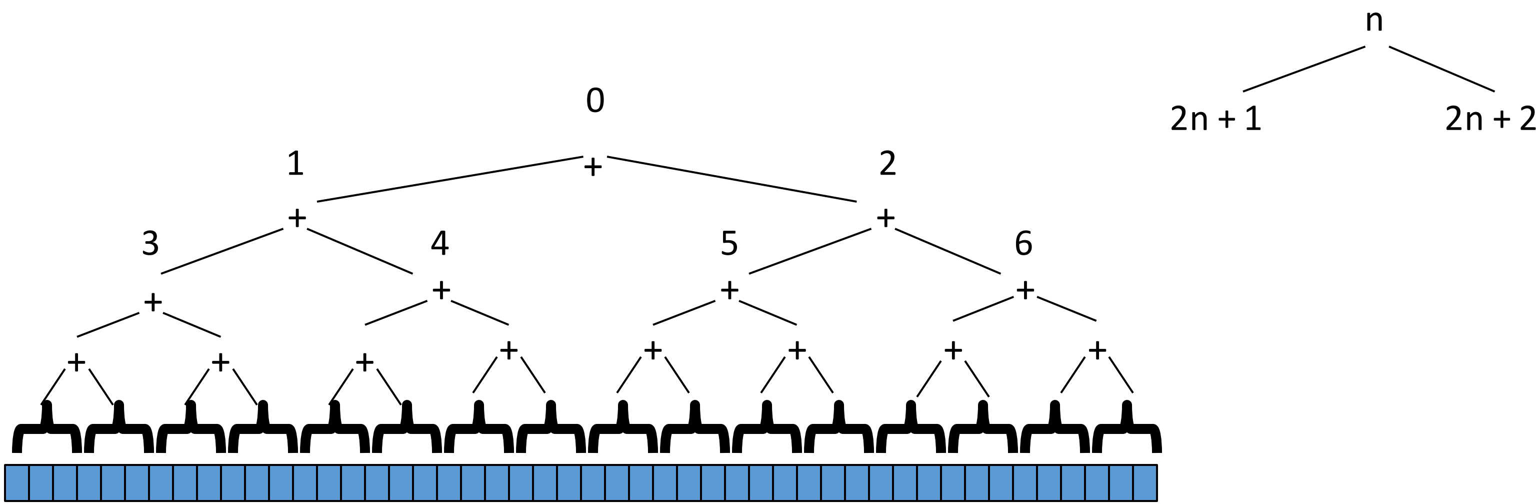


Task B:

Extend your implementation such that it creates only a fixed number of threads. Make sure that your solution is properly synchronized when checking whether to create a new thread

How to achieve this?

Divide and Conquer Parallelization



Option 1:
Shared counter with synchronized/atomic access

Option 2:
Assign unique sequential id to each task. Spawn threads for first N tasks.

+ no synchronization required
- imbalanced amount of work

Alternative approach (from homework submissions)

Instead of using ids, we give the child tasks „numThreads / 2“ since that‘s the amount of threads that are allowed to be created in the subtrees.

Let‘s take a look at the master solution

Task D

Implement it with ExecutorService

Let‘s take a look at the master solution.

ExecutorService

TPS01-J. Do not execute interdependent tasks in a bounded thread pool

Created by Dhruv Mohindra, last modified by Carol J. Lallier on Jun 22, 2015

Bounded thread pools allow the programmer to specify an upper limit on the number of threads that can concurrently execute in a thread pool. Programs must not use threads from a bounded thread pool to execute tasks that depend on the completion of other tasks in the pool.

A form of deadlock called *thread-starvation deadlock* arises when all the threads executing in the pool are blocked on tasks that are waiting on an internal queue for an available thread in which to execute. Thread-starvation deadlock occurs when currently executing tasks submit other tasks to a thread pool and wait for them to complete and the thread pool lacks the capacity to accommodate all the tasks at once.

This problem can be confusing because the program can function correctly when fewer threads are needed. The issue can be mitigated, in some cases, by choosing a larger pool size. However, determining a suitable size may be difficult or even impossible.

Similarly, threads in a thread pool may fail to be recycled when two executing tasks each require the other to complete before they can terminate. A blocking operation within a subtask can also lead to unbounded queue growth [Goetz 2006].

Fork/Join: recommended for Divide and Conquer tasks as they have strong task interdependency

ExecutorService: for handling many independent requests where tasks are standalone

Divide and Conquer vs Fork/Join

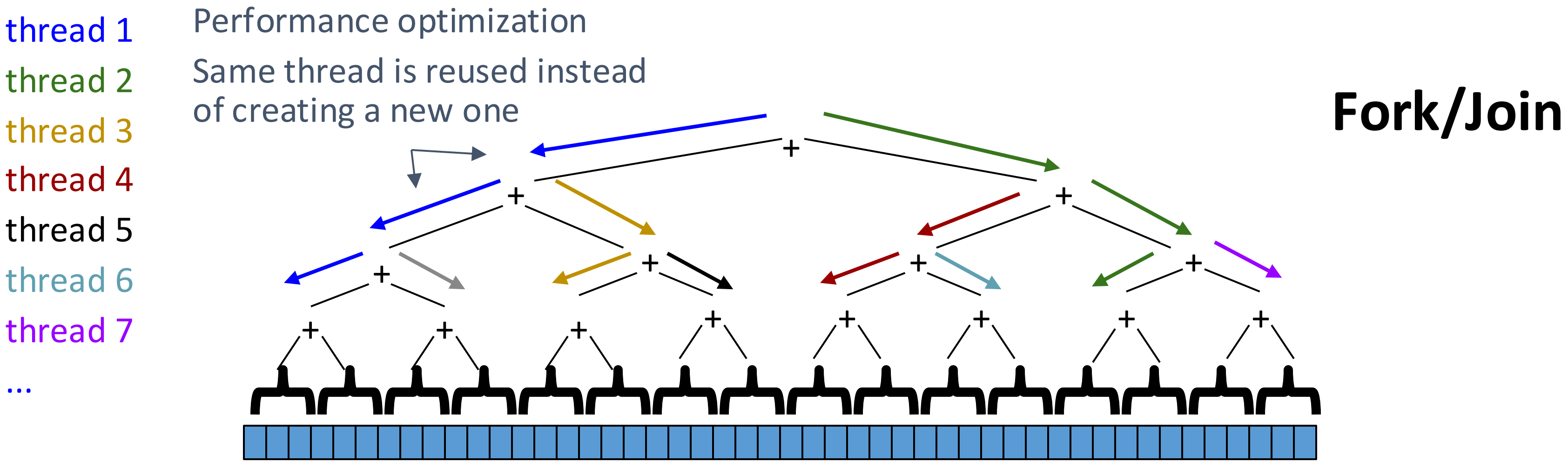
Divide And Conquer

Fundamental design pattern based on recursively breaking down a problem into smaller problems that can be combined to give a solution to the original problem

Fork/Join

A framework that supports Divide and Conquer style parallelism

Divide and Conquer vs Fork/Join



a framework that supports Divide and Conquer style parallelism

problems are solved in parallel

Search And Count

```java
public class SearchAndCountMultiple
  extends RecursiveTask<Integer> {
        private int[] input;
        private int start;
        private int length;
        private int cutOff;
        private Workload.Type type;
}
```

```java
protected Integer compute() {
  if (// work is small)

    // do the work directly


  else {
    // split work into pieces





    // invoke the pieces and
       wait for the results



    // combine the results
  }
}
```

```java
protected Integer compute() {
  if (length <= cutOff) {
    int count = 0;
    for (int i = start; i < start + length; i++) {
      if (Workload.doWork(input[i], type)) count++;
    }
    return count;
  else {
    int half = (length) / 2;
    SearchAndCountMultiple sc1 =
      new SearchAndCountMultiple(input, start, half, cutOff, type);
    SearchAndCountMultiple sc2 =
      new SearchAndCountMultiple(input, start + half, length - half, cutOff, type);

    sc1.fork();
    sc2.fork();
    int count1 = sc1.join();
    int count2 = sc2.join();
    return count1 + count2;
  }
}
```

Theory

Lock Object

Shared object that satisfies the following interface

```
public interface Lock{
        public void lock();      // entering CS
        public void unlock();    // leaving CS
}
```

providing the following semantics

new Lock make a new lock, initially *"not held"*

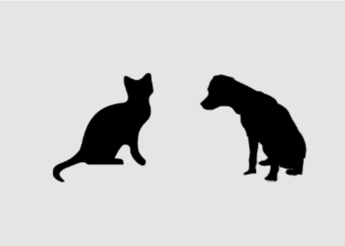
acquire blocks (only) if this lock is already currently *"held"*
Once *"not held"*, makes lock *"held"* [all at once!]

release makes this lock *"not held"*
If >= 1 threads are blocked on it, exactly 1 will acquire it

Required Properties of Mutual Exclusion

Safety Property

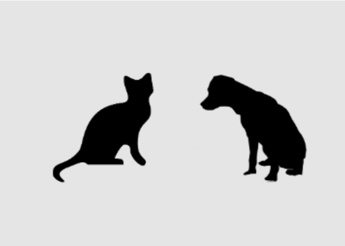
- At most one process executes the critical section code



Required Properties of Mutual Exclusion

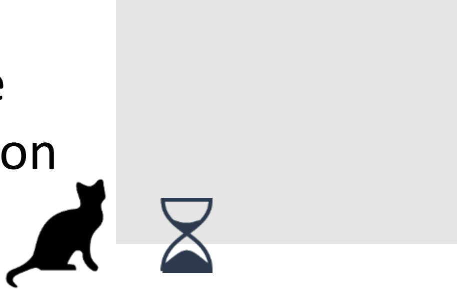
Safety Property

- At most one process executes the critical section code



Liveness

- *Minimally*: acquire_mutex must terminate in finite time when no process executes in the critical section



Almost-correct pseudocode

```
class BankAccount {
   private int balance = 0;
   private Lock lk = new Lock();
   …
   void withdraw(int amount) {
      lk.lock(); // may block
      int b = getBalance();
      if(amount > b)
         throw new WithdrawTooLargeException();
      setBalance(b – amount);
      lk.unlock();
   }
   // deposit would also acquire/release lk
}
```

One lock for each account

Almost-correct pseudocode

```
class BankAccount {
   private int balance = 0;
   private Lock lk = new Lock();
   …
   void withdraw(int amount) {
      lk.lock(); // may block
      int b = getBalance();
      if(amount > b)
         throw new WithdrawTooLargeException();
      setBalance(b – amount);
      lk.unlock();
   }
   // deposit would also acquire/release lk
}
```

One lock for each account

Lock won't be released if exception is thrown!

Solution: Use try/finally block!

```java
Lock lk = new ReentrantLock();

public static long criticalWork() {
  lk.lock();
  try {
    //do some work
    return result;
  } finally {
    lk.unlock();
  }
}
```

Always gets executed
(even after exception
or return)

Possible mistakes

Incorrect: Use different locks for **`withdraw`** and **`deposit`**

- Mutual exclusion works only when using same lock
- **`balance`** field is the shared resource being protected

Poor performance: Use same lock for every bank account

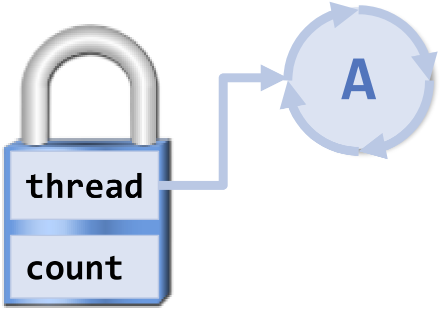
- No simultaneous operations on different accounts

Incorrect: Forget to release a lock (blocks other threads forever!)

- Previous slide is wrong because of the exception possibility!

```
if(amount > b) {
  lk.unlock(); // hard to remember!
  throw new WithdrawTooLargeException();
}
```

Re-entrant lock



A **re-entrant lock** (a.k.a. **recursive lock**)

“remembers”

- the thread (if any) that currently holds it
- a *count*

When the lock goes from held to not-held, the count is set to 0

If (code running in) the current holder calls **`lock(acquire)`**:

- it does not block
- it increments the count

On **`unlock(release)`**:

- if the count is > 0, the count is decremented
- if the count is 0, the lock becomes *not-held*

Re-entrant locks work

- This simple code works fine provided **`lk`** is a reentrant lock
- Okay to call **`setBalance`** directly
- Okay to call **`withdraw`** (won't block forever)

```
int setBalance(int x) {
  lk.lock();
  balance = x;
  lk.unlock();
}

void withdraw(int amount) {
  lk.lock();
  …
  setBalance(b – amount);
  lk.unlock();
}
```

Race condition

A **Race Condition** occurs in concurrent programming when the correctness of the system depends on the specific interleaving or ordering of operations executed by multiple threads or processes.

Typically, problem is some *intermediate state* that "messes up" a concurrent thread that "sees" that state

Note: This lecture makes a big distinction between *data races* and *bad interleavings*, both instances of race-condition bugs

- Confusion often results from not distinguishing these or using the ambiguous "race condition" to mean only one

The distinction

Data Race [aka *Low Level Race Condition, low semantic level*]
Erroneous program behavior caused by insufficiently synchronized accesses of a shared resource by multiple threads, e.g. Simultaneous read/write or write/write of the same memory location

(for mortals) **always** an error, due to compiler & HW

Bad Interleaving [aka *High Level Race Condition, high semantic level*]
Erroneous program behavior caused by an unfavorable execution order of a multithreaded algorithm that makes use of otherwise well synchronized resources.

“Bad” depends on your specification

The prefix-sum problem

Given **`int[] input`**,

produce **`int[] output`** where:

`output[i]` = **`input[0]+input[1]+…+input[i]`**

Sequential prefix-sum

```
int[] prefix_sum(int[] input){
    int[] output = new int[input.length];
    output[0] = input[0];

    for(int i = 1; i < input.length; i++)
        output[i] = output[i-1] + input[i];

    return output;
}
```

Does not seem parallelizable

- Work: O(n), Span: O(n)
- This algorithm is sequential, but a **different algorithm** has: Work O(n), Span O(log n)

Example

How to compute the prefix-sum in parallel?

input	6	4	16	10	16	14	2	8
output	6	10	26	36	52	66	68	76

Example

`input`	6	4	16	10	16	14	2	8
`output`								

Example

`input`	6	4	16	10	16	14	2	8
`output`	6	10	26	36	16	30	32	40

Example

+36

input	6	4	16	10	16	14	2	8
output	6	10	26	36	52	66	68	76

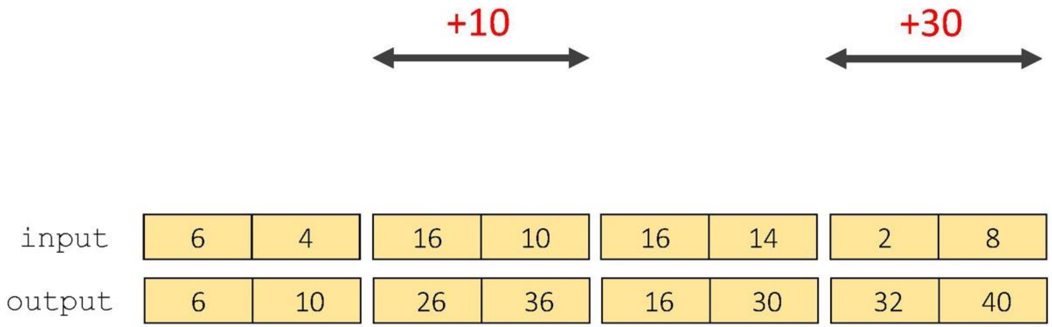
Example

`input`	6	4	16	10	16	14	2	8
`output`	6	10	16	26	16	30	2	10

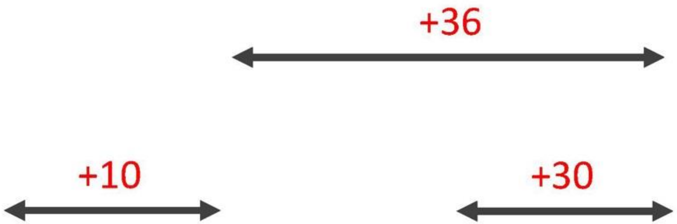
Example

`input`	6	4	16	10	16	14	2	8
`output`	6	10	26	36	16	30	2	10

Example



Example



input	6	4	16	10	16	14	2	8
output	6	10	26	36	52	66	68	76

Parallel prefix-sum

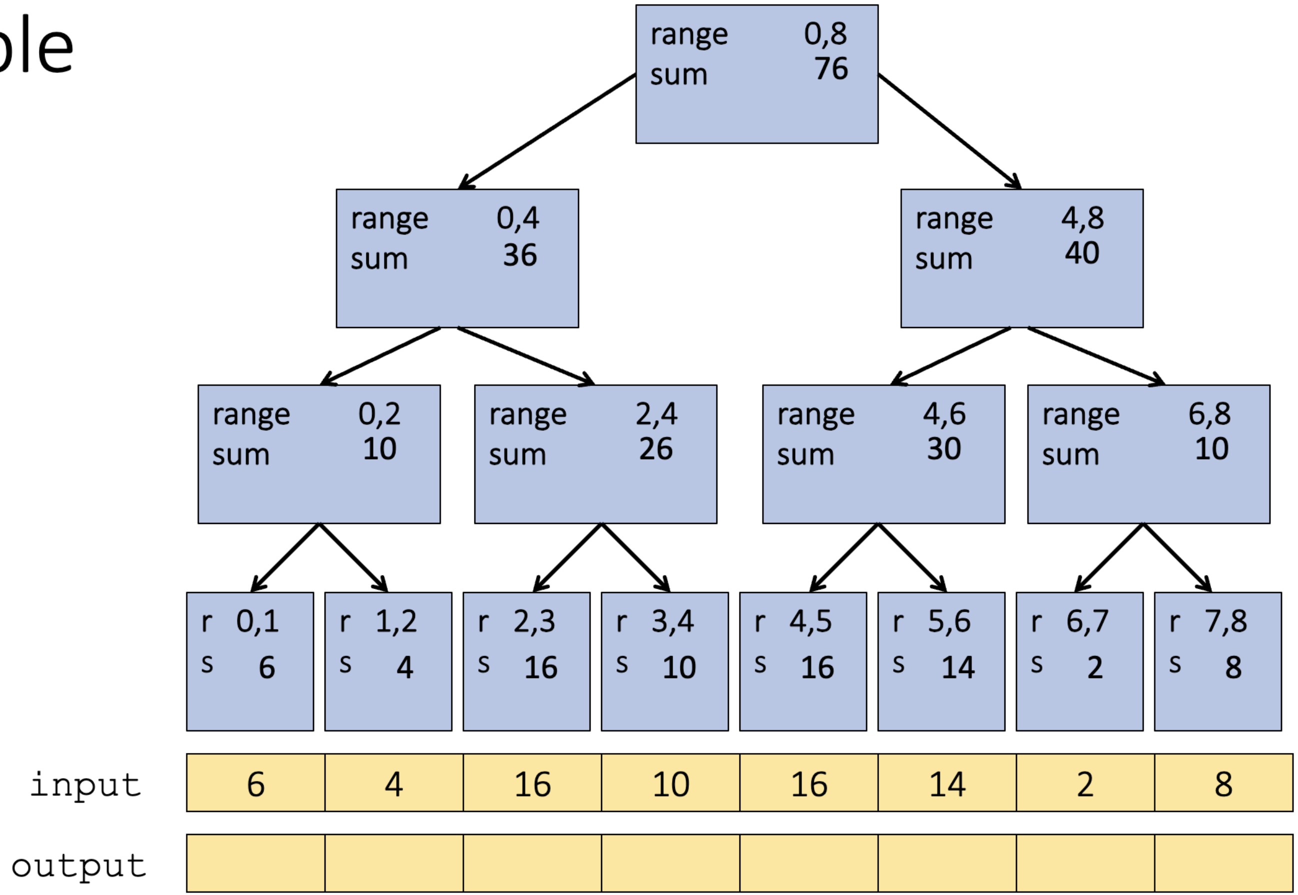
The parallel-prefix algorithm does two passes

- Each pass has O(n) work and O(`log` n) span
- So in total there is O(n) work and O(`log` n) span
- So like with array summing, parallelism is n/`log` n

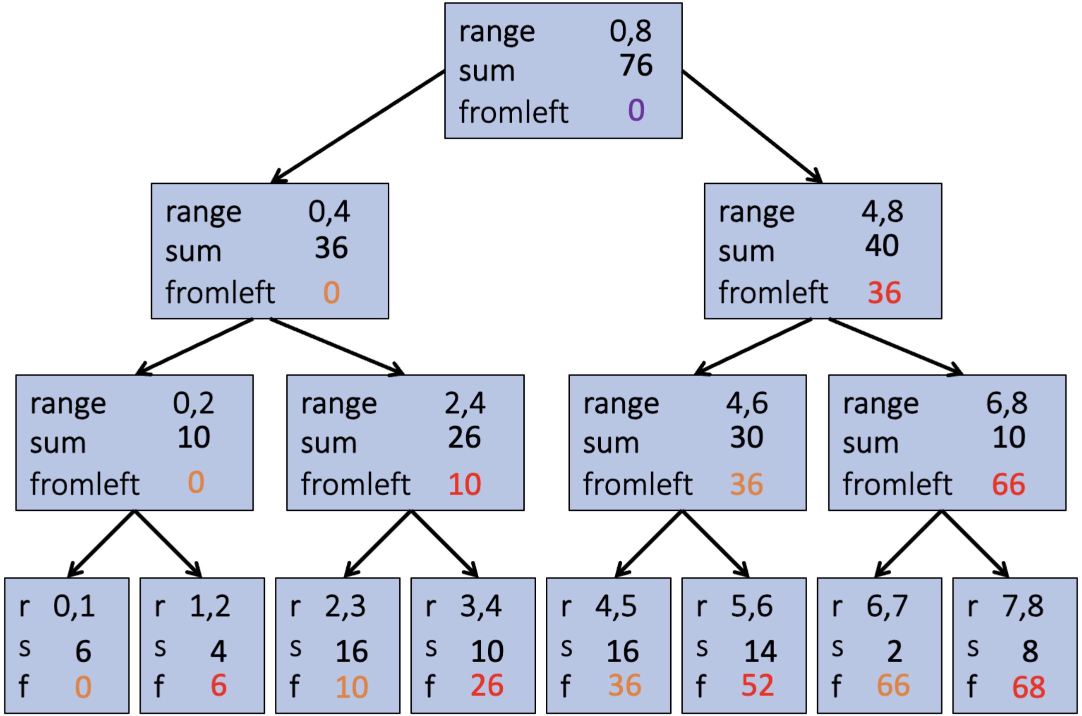
First pass builds a tree bottom-up: the “up” pass

Second pass traverses the tree top-down: the “down” pass

Example


range 0,8
sum 76
range 0,4
sum 36
range 4,8
sum 40
range 0,2
sum 10
range 2,4
sum 26
range 4,6
sum 30
range 6,8
sum 10
r 0,1
s 6
r 1,2
s 4
r 2,3
s 16
r 3,4
s 10
r 4,5
s 16
r 5,6
s 14
r 6,7
s 2
r 7,8
s 8
input
6
4
16
10
16
14
2
8
output

Example



input	6	4	16	10	16	14	2	8
output	6	10	26	36	52	66	68	76

Parallel Patterns

- We are now quite familiar with how to parallize algorithms
- There are a few recurring patterns that are important to know

Map, Reduction, Stencil, Scan, Pack

Reduction

- A reduction is an operation that produces a single answer from a collection (array etc) via an **associative** operator.
- Needs to be associative. Otherwise divide-and-conquer won‘t work

Example: array sum

Map

- Operates on each element of the input data indenpendently (each array element)
- Output is the same size → no size reduction
- Doesn‘t have to be the same operation on each element

Example: add two arrays

Stencil

- Like map but can take more than one element as input
- Generalization of map and thus also no size reduction

Example:

Image → apply averaging filter on each pixel

Update a value based on its neighbors

Never do it in-place because you would then take values that are already output values.

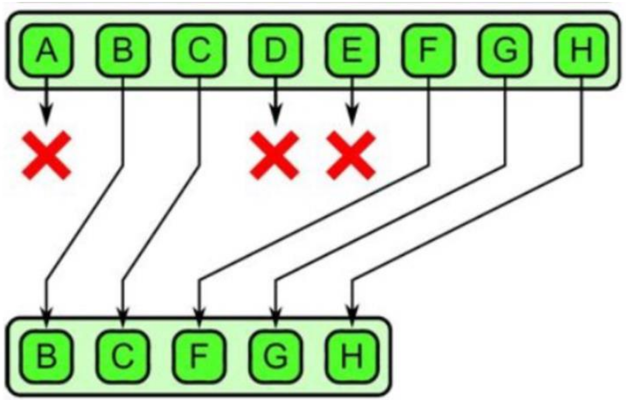
Scan

- Collection of data X → return collection of data Y
- Y(i) = functionOf(Y(i - 1) & X(i))
- Seems sequential because of dependencies
- Can parallelize if function is associative → O(log(n)) span

Example: parallel prefix sum

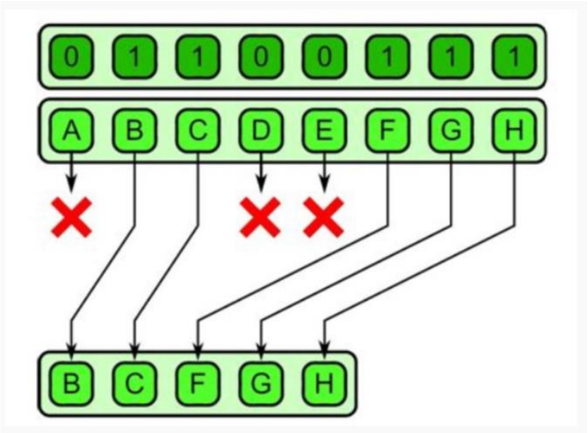
Pack

- Collection of data X → return collection of data X if fulfill condition



Pack

- First compute bit vector
- Then find index in result array (prefix sum on bit vector)



Pre-Discussion
Exercise 6

Assignment 6

Task Parallelism:

- Merge Sort
- Longest Sequence

Merge sort algorithm

In this exercise you will implement the merge sort algorithm using task parallelism.

The merge sort algorithm partitions the array into smaller arrays, sorts each one separately and then merges the sorted arrays.

- By default, the partitioning of the array continues recursively until the array size is 1 or 2, which then is sorted trivially.
- Try larger cutoff values (e.g g partition arrays down to minimum size 4 instead of 2) and see how this affects the algorithm performance.
- Discuss the asymptotic running time of the algorithm and the obtained speedup.

Longest Sequence

Given a sequence of numbers:

[1, 9, 4, 3, 3, 8, 7, 7, 7, 0]

find the longest sequence of the same consecutive number.

If multiple sequences have the same length, return the first one (the one with lowest starting index)

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

[1, 1, 0, 0]

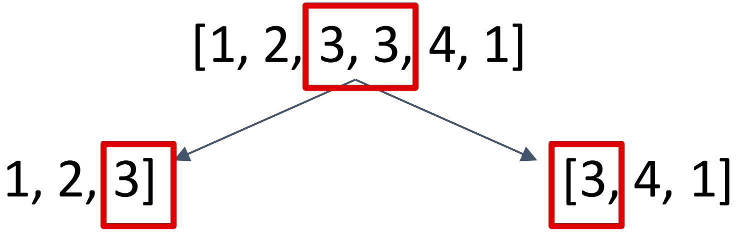
Longest Sequence

Task:

Implement task parallel version that finds the longest sequence of the same consecutive number.

Challenge:

The input array cannot be divided arbitrarily. For example:



Combining results of subtasks does not give the correct answer!

Kahoot!